

## Exponential Growth and Decay

### Exponential growth and living things

Populations of living things can increase exponentially under certain circumstances. For example, scientists often grow bacteria in a *Petri dish*. A Petri dish is a glass container with a lid. A gelled nutrient solution is put in the bottom. The scientist places a sample of bacteria in the Petri dish to see how it grows under different conditions. Placing bacteria in a growth medium is called *inoculation*.



Bacteria growing in a Petri dish.

Photo courtesy Limnology Lab and Field

When a growth medium is inoculated with bacteria, there is little competition for nutrients. With unlimited food, the bacteria can then grow at an unlimited rate. Let's say, for example, that each bacterium in a Petri dish divides into two each hour.

1. If there are 100 bacteria in the dish when the dish is inoculated, how many bacteria will be in the dish after 1 hour? \_\_\_\_\_. How many will there be after 2 hours?  
\_\_\_\_\_
2. Build a table for the number of bacteria present in the Petri dish as a function of time.

Hour	0	1	2	3	4	5	6
Bacteria Count	100						

When populations grow at an unlimited rate, they exhibit *exponential growth*. The word 'exponential' has the same root as the word 'exponent'. We call the growth exponential because it can be closely modeled with an exponential function.

An *exponential function* has the general form  $f(x) = a \cdot b^x$ . The  $a$  in the equation is the initial value, or initial population. In this example, it is how many bacteria are inoculated into the Petri dish.

3. What is the initial value in the example above? \_\_\_\_\_

The  $b$  in the equation is the common ratio, or the ratio between the bacteria count in one hour and the next.

4. What is the common ratio in the example above? \_\_\_\_\_
5. Now put the initial value and common ratio into the exponential function. Write the exponential function here. \_\_\_\_\_

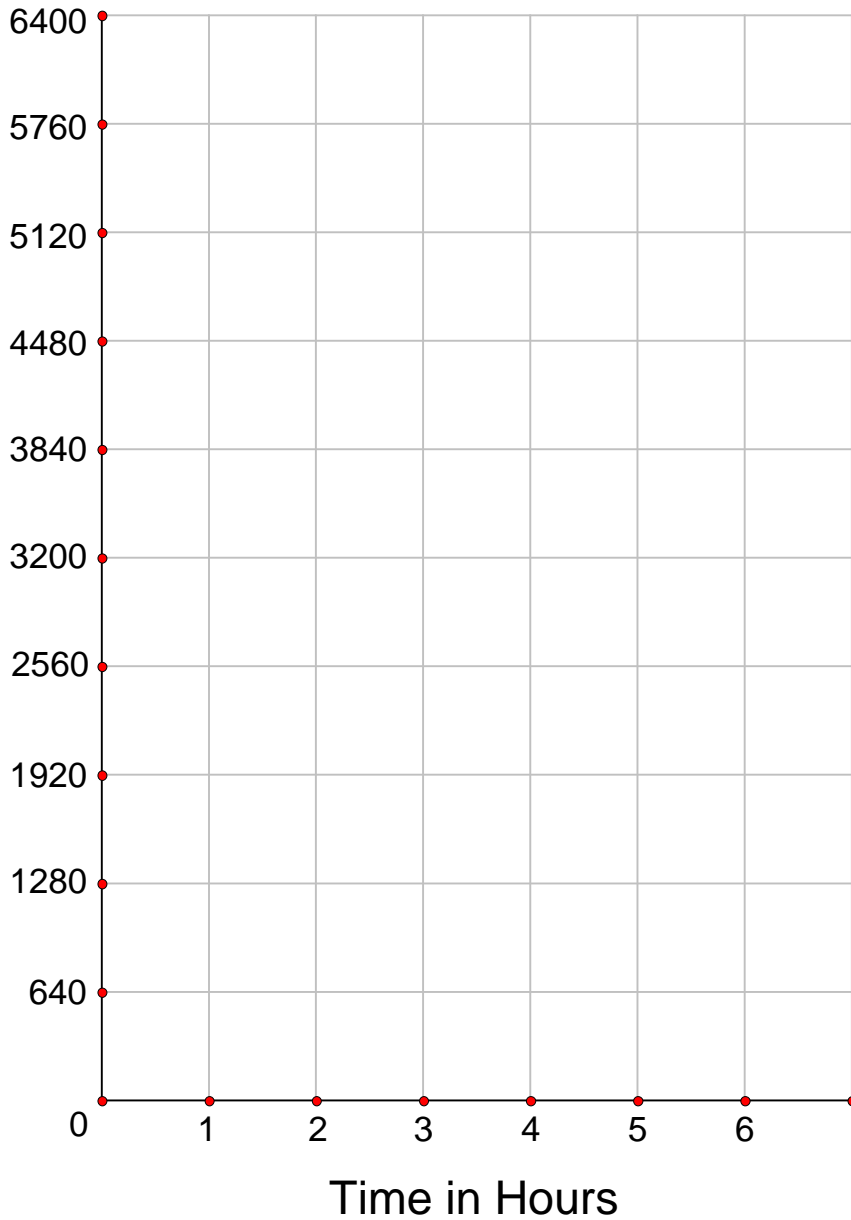
Name \_\_\_\_\_ Period \_\_\_\_\_

6. Build a table using the exponential function. Does it match the table in question 2?

Hour	0	1	2	3	4	5	6
Bacteria count	$f(0) = 100$						

7. The numbers in the two tables should match exactly. Now graph the exponential function you created.

### Bacteria Count



## Exponential Decay and Folding Paper

Exponential growth involves something getting larger by a common ratio. Exponential decay involves something getting smaller by a common ratio. We will be using a blank piece of paper to explore exponential decay and how it is related to exponential growth.

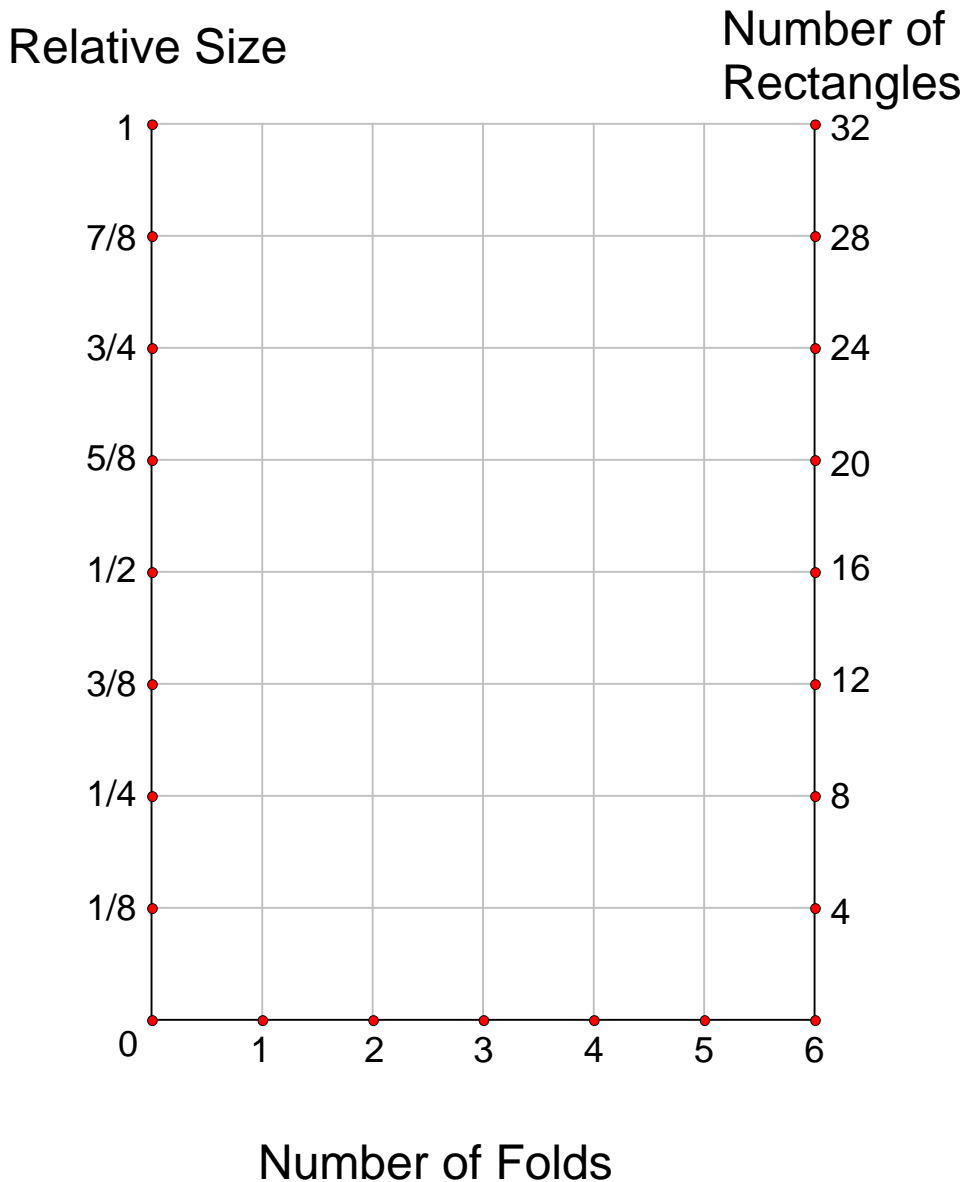
You will be folding a piece of paper in half multiple times. You will fill in the table below as you do this experiment. Each time you fold the paper, fill in the relative size of the paper to the original. For example, when you fold the paper in half, its relative size is  $\frac{1}{2}$  of the original. Make sure to use fractions, not decimals. Also fill in the number of rectangles into which the paper is now divided.

8.

Fold Number	0	1	2	3	4	5
Relative Size	1	$\frac{1}{2}$				
Num Rectangles	1	2				

What do you notice about the relative size and number of rectangles? The relative size and rectangles are multiplicative inverses of each other.

9. Now graph the data from table on the graph below. Use the scale on the right hand side for relative size, and the scale on the left hand side for the number of rectangles.



10. Take a look at the two graphs. What do they have in common? \_\_\_\_\_  
 If you swapped the vertical axis and put relative size on the right and number of rectangles on the left, would the lines look any different? \_\_\_\_\_
11. (extra credit) When one curve on a graph can be turned into another by flipping it around a line (in this case, number of folds=3), the curves are called reflexive. Why can an exponential growth curve be reflexive to an exponential decay curve?

Name \_\_\_\_\_ Period \_\_\_\_\_

## Glossary

exponential decay Exponential decay occurs when something is reduced by a ratio each time period.

exponential function An exponential function is a function of the form  $y = a \cdot b^x$

exponential growth Exponential growth occurs when something increases by a ratio each time period.

inoculation Inoculation is when bacteria, mold, fungus or other living thing is placed in a growth medium.

Petri dish A Petri dish is a round, flat bottomed glass dish with a cover. It is used for growing bacteria, molds, and funguses.