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Intermediate Algebra – Properties of Exponents Worksheet

Properties of exponents

$b^m = b_1 \cdot b_2 \cdot b_3 \cdot \ldots \cdot b_m$

$b^m \cdot b^n = b^{m-n}$

$\frac{1}{b^m} = b^{-m}$

$\frac{b^m}{b^n} = b^{m-n}$

$b^0 = 1,\ b \neq 0$

$\left(b^m\right)^n = b^{m \cdot n}$

$\sqrt[n]{b} = b^{1/n}$

$\sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m = b^{m/n}$

Definition of Exponents

An exponent is defined as meaning multiplying a value by itself a certain number of times. For example, $b^2 = \underline{b \cdot b}$, or b times itself twice. Expand the following exponents:

1. $b^2 = \underline{b \cdot b}$
2. $g^5 =$
3. $s^4 =$
4. $r^3 =$
5. $z^6 =$
6. $5^{15} =$

Definition of Negative Exponents

A negative exponent is defined as division. For example, $b^{-3} = \frac{1}{\underline{b \cdot b \cdot b}}$. Expand the following negative exponents.

Ex. $5^{-2} = \frac{1}{5 \cdot 5 \cdot 5}$

7. $t^{-2} =$
8. $g^{-4} =$
9. $6^{-4} =$
10. $m^{-6} =$
11. $p^{-14} =$
12. $d^3 =$

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Product Property of Exponents

When we multiply like bases, we add the exponents. For example, $b^3 \cdot b^5 = b^{3+5} = b^8$. We can show this by using the definition of an exponent, and the properties of real numbers. We can use the associative property of multiplication to write $b^3 \cdot b^5$ as $(b^3) \cdot (b^5)$. Using the definition of exponents, we write $(b \cdot b \cdot b) \cdot (b \cdot b \cdot b \cdot b \cdot b)$. Then we apply the associative property of multiplication again to get $b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b$. Finally, we use the definition of exponents to get b^8. Simplify the following exponents using the product property of exponents. Note: Since $r^3 + r^4$ has addition in the middle and NOT multiplication, it can NOT be simplified.

Ex: $b^3 \cdot b^5 = b^{3+5} = b^8$

13. $a^5 \cdot a^7$
14. $b^3 + b^4$
15. $t^{-2} \cdot t^3$
16. $r^2 \cdot r^3$
17. $(1+7)^2$
18. $p^{-4} \cdot p^{-2}$

Quotient Property of Exponents

The Quotient Property of Exponents says that $\frac{b^m}{b^n} = b^{m-n}$. We can show this using the definition of negative exponents. $\frac{b^m}{b^n} = \frac{b^m}{1} \cdot \frac{1}{b^n} = b^m \cdot \frac{1}{b^n}$. Now we apply the Property of Negative Exponents. $b^m \cdot \frac{1}{b^n} = b^m \cdot b^{-n}$. Then we apply the Product Property of Exponents: $b^m \cdot b^{-n} = b^{m-n}$.

Ex: $\frac{t^4}{t^2} = t^{4-2} = t^2$

1. $\frac{s^5}{s^2}$
2. $\frac{z^2}{z_4}$
3. $\frac{m^{-3}}{m^2}$
4. $\frac{5^2}{5^3}$
5. $\frac{v^x}{v^y}$
6. $\frac{m^{-5}}{m^{-6}}$

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Property of the Zero Exponent

When we have an exponent of zero, the result is always one. Here is why. We know that anything except 0, when divided by itself is 1. So $\frac{b^m}{b^m}=1, b \neq 0$. Continuing with the Quotient Property of Exponents, $\frac{b^m}{b^m}=b^{m-m}$. Using arithmetic, we get $b^{m-m}=1$. So we can conclude that $b^0=1, b \neq 0$. Simplify the following using the Property of the Zero Exponent.

Ex: $(a+b)^0=1$

7. 5^0
8. b^{3-3}
9. $\left(r+2t-z^7\right)^0$
10. $(j^u s^t \cdot o \cdot n^e)^0$

Exponents and Roots.

Originally, roots were written with the radical ($\sqrt{}$). Later, as the discipline of mathematics progressed, mathematicians realized that a root could be expressed with an exponent. For example, $\sqrt{3}=3^{1/2}$ and $\sqrt[3]{5}=5^{1/3}$. Convert the following roots to exponents.

Ex: $\sqrt[3]{5}=5^{1/3}$

11. $\sqrt{g^2}$
12. $\sqrt{17}$
13. $\sqrt{x}$
14. $\sqrt[3]{r}$
15. $\sqrt{p^3}$
16. $\sqrt[4]{a+b}$

Convert the following exponents to roots.

Ex: $u^{3/5}=\sqrt[3]{u^5}$

17. $x^{1/2}$
18. $f^{1/3}$
19. $h^{2/3}$
20. $e^{1/y}$
21. $z^{x/y}$
22. $q^{-1/3}$

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Exponents to a Power

We can also raise an exponent to another exponent. It looks like this: $\left(e^3\right)^2 = e^{2\cdot 3} = e^6$. Note that this is NOT the same as e^{3^2}, which is equal to e^9. When raising an exponent to a power, we multiply the exponents. Here is why: Start with $\left(e^2\right)^3$. Using the order of operations, we simplify inside the parenthesis first. We will use the definition of exponents. $\left(e^2\right)^3 = (e\cdot e)^3$. Now we apply the definition of exponents again. $(e\cdot e)^3 = (e\cdot e)\cdot(e\cdot e)\cdot(e\cdot e)$. Now we use the associative property of multiplication. $(e\cdot e)\cdot(e\cdot e)\cdot(e\cdot e) = e\cdot e\cdot e\cdot e\cdot e\cdot e$. Finally we apply the definition of exponents $e\cdot e\cdot e\cdot e\cdot e\cdot e = e^6$. Simplify the following expressions using the power of exponents.

Ex: $\left(e^3\right)^2 = e^{2\cdot 3} = e^6$

23. $\left(k^3\right)^5$
24. $w^{(2)^3}$
25. $\left(r^3\right)^5$
26. $\left(p^2\right)^x$
27. $\left(v^t\right)^2$
28. $\left(v^7\right)^7$

Put it All Together

Simplify the following expressions using the properties of exponents.

Ex: $\left(\dfrac{j^3}{j^{-2}}\right)^2 = \left(j^{3-(-2)}\right)^2 = \left(j^5\right)^2 = j^{5\cdot 2} = j^{10}$

29. $\sqrt{\dfrac{18}{2}}$
30. $\sqrt{25}$
31. $\left(k^3 \cdot k^5\right)^2$
32. $\dfrac{r^3 \cdot p^2}{r^2}$
33. $\dfrac{r^2}{z^3}$
34. $\dfrac{h^{-2}}{h^{-3}}$
35. $\left(3^2\right)^5$
36. $\sqrt{r^4}$
37. $\sqrt[2]{t^5}$
38. $\sqrt[3]{j^9}$
39. $m^5 \cdot m^2 + m^3$
40. $\sqrt{\dfrac{u^5}{u^3}}$